lines are weak compared to the strong lines of the Uviarc spectrum. The scattered radiation when the monochromator is set at one of the stronger mercury lines is less than 1 percent. After the mirrors were realuminized the scattered radiation in the 2301A line was 5 percent and about 0.3 percent for the stronger mercury lines.

After the second monochromator was in operation, a third mirror of 12-foot radius of curvature was added so that there might be an alternative arrangement in which collimator and telescope lengths were doubled, reducing distortion and improving resolution as well as permitting the use of longer sources. These advantages are gained by the change in mirrors, but not to a sufficient degree that this alternative arrangement has proved useful. The original choice of dimensions has proved to be in many ways a fortuitous combination, the distortion being just at the threshold of decreasing resolution.

ANOTHER FREE-PRISM MONOCHROMATOR

For a free-surface prism the problems of distortion and odd beam angle can be largely met by resorting to three reflections. A single concave mirror serves as collimator and telescope in two separate reflections, and a front-surface piece of plate glass immersed slantwise in a tank of water acts as the third reflecting surface. This arrangement has been set up to test and is in many respects superior to the Harrison-type prism, although because of the three reflections it is not likely to be useful below 2400A. However, at higher wavelengths where aluminum is an efficient reflector the arrangement has the advantages of horizontal beam (and slits), a simple wavelength adjustment by tilting the underwater mirror, fixed beam direction and position, and increase of dispersion toward longer wavelength settings. Various combinations of aperture and resolution may be set up by adjustment of the two mirrors, and it has been possible to resolve bands of better than 10A half-width at 2800A. The only special parts required are a front-surfaced concave mirror such as amateur astronomers use and a front-surfaced piece of plate glass. The mirror used for trial of this arrangement is 10 inches in diameter and spherically figured to a 12-foot radius and cost about $100. The arrangement appears to be a simple and effective one for monochromatic work in the region of maximum nucleic acid absorption, around 2600A.

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New Contributions to the Optics of Intensely Light-Scattering Materials. Part II: Nonhomogeneous Layers*

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The derived laws apply to layers whose scattering coefficient S and absorption coefficient K vary vertically to the surface of the layer. In the general case the differential equations of the preceding paper [P. Kubelka, J. Opt. Soc. Am. 38, 448 (1948)] must be used; the coefficients, however, hitherto constant, now are functions of the distance x from the surface. In the practically important case in which K/S is constant, one may introduce the variable p, such that $p \equiv \int_0^x S(x)dx$. One reduces thereby the nonhomogeneous to the previously treated homogeneous case.

Transmittance $T_{1,2}$ and reflectance $R_{1,2}$ of two nonhomogeneous sheets can be calculated by the following equations:

$$T_{1,2}=\frac{T_1T_2}{1-R_IR_2}, \qquad R_{1,2}=R_1+\frac{T_1^2R_2}{1-R_IR_2},$$

where T_1, T_2, R_1, R_2 are the transmittances and reflectances of the single sheets, and R_I represents the reflectance of the first sheet when illuminated in the inverse direction. Analogous formulas for more sheets and formulas relating transmittance, reflectance for specimens upon black, gray or white backing surfaces, and contrast ratio, are derived.

It is shown by theory and experiment that reflectance and absorption of a nonhomogeneous specimen depend on the direction of illumination, whereas transmittance does not.

INTRODUCTION

IN two previous papers[1,2] the optical behavior of light-scattering materials has been treated theoretically. The materials were assumed to be "homogeneous in this way that the optical inhomogeneities (necessarily existing when the light shall be scattered) are incom-

* An abstract of this work has been presented to the Brazilian Academy of Science in a lecture given by the author in the session of December 28, 1949.

[1] Paul Kubelka and Franz Munk, Z. tech. Phys. 12, 593 (1931). See also abstracts in English given by F. A. Steele, reference 5; Deane B. Judd, reference 6; H. A. Gardner, *Physical and Chemical Examination of Paints, Varnishes, Laquers and Colors* (Inst. Paints Varnishes Research, Washington, D. C., 1939), ninth edition, p. 10*ff*.

[2] Paul Kubelka, J. Opt. Soc. Am. 38, 448 (1948).

parably smaller than the thickness of the specimen and uniformly distributed in the material." We now weaken this restriction, in so far as we shall deal with specimens varying as to the optical properties of their material from one surface to the other (the surfaces being—as assumed in the previous investigations—plane, parallel and infinitely extended). On the other hand, the optical properties may not vary along planes parallel to the surfaces. The incident light is supposed, once more, to be diffused and to remain unchanged in its angular and wavelength distribution by reflection or transmission. The medium in which the scattering material is embedded is again assumed to be the same as the medium of incidence (generally air).

This last assumption will be nearly realized, when we deal e.g. with dull paper or with dull waterpaints. If we assume the medium to be air, we may ignore reflection at the surface of the specimen. In a recent work, Stenius[3] determined the influence of surface reflection of paper with regard to the author's theory[1,2] on homogeneous layers (which started from the same assumption). He came to the conclusion that surface reflections of dull paper are negligible. This fact also follows from earlier investigations of Steele[4] and Judd and his collaborators.[5]

I. THE GENERAL CASE

The following system of differential equations has been derived in the previous papers[1,2] for a homogeneous light-scattering material:

$$\begin{aligned} -di &= -(S+K)idx+Sjdx, \\ dj &= -(S+K)jdx+Sidx \end{aligned} \tag{1}$$

($i \equiv$ flux of the diffused light traveling inside the specimen toward its unilluminated surface, at the distance x from this surface; $j \equiv$ flux of the light traveling in the inverse direction; $S \equiv$ coefficient of scatter; $K \equiv$ coefficient of absorption.) These equations hold also for nonhomogeneous specimens. S and K, however, hitherto constant, become functions of x.

In the general case we introduce the functions $S=S(x)$ and $K=K(x)$. By integrating Eqs. (1) between $x=0$ and $x=X$ (X thickness of the specimen), we may deduce equations for the transmittance T and the reflectance R of the specimen. It will, however, not often happen that the functions $S=S(x)$ and $K=K(x)$ are known. Therefore we shall limit our considerations to several special cases.

II. THE CASE K/S=CONSTANT

When a homogeneous scattering phase (e.g. cellulose) is nonuniformly dispersed in a colorless medium (e.g. air, in the case of paper), S and K are very often proportional to each other, so that K/S and $(S+K)/S$ are constant, whereas S is variable. We may then introduce into Eqs. (1) the new variable p such that

$$p=\int_0^x S(x)dx, \tag{2}$$

whence

$$S(x)dx=dp. \tag{3}$$

We call p the scattering power of the layer $x=0$ to $x=x$. The scattering power P of the whole specimen of thickness X is given by

$$P=\int_0^X S(x)dx. \tag{4}$$

The concept of "scattering power" has been introduced in the literature by Judd.[5] It is extended here so as to apply to nonhomogeneous layers. Hence it becomes particularly useful. The differential equations now read

$$\begin{aligned} -di &= -(S+K/S)idp+jdp, \\ dj &= -(S+K/S)jdp+idp. \end{aligned} \tag{5}$$

These equations are not only now more easily integrated but also lead to the same formulas as did the original equations 1 in which S and K were constant. We may adopt all the formulas complied in the previous paper,[2] including the various approximations. We only have to exchange the special value of scattering power SX, which is valid for homogeneous specimens, by the general value, symbolized by the letter P. For instance

Reflectance: $$R=\frac{1}{a+b\operatorname{ctgh}bP}; \tag{6}$$

Transmittance: $$T=\frac{b}{a\sinh bP+b\cosh bP}; \tag{7}$$

$$a\equiv(S+K)/S, \quad b\equiv(a^2-1)^{\frac{1}{2}}.$$

We may now introduce the content $D(x)$ of the disperse phase per unit volume. S and K will be proportional to D if the disperse phase itself is homogeneous, as assumed. Consequently

$$a\equiv(S+K/S)=\text{constant}, \quad S(x)=S_wD(x),$$

S_w being a constant which characterizes not the turbid material but the dispersed phase as to its ability to scatter light. When the medium is air, D is practically identical with the density of the concerned layer. So we find [Eq. (4)]:

$$P=\int_0^X S(x)dx=S_w\int_0^X D(x)dx=S_wX_w,$$

X_w being the weight of one unit square of the specimen. Now there exists full analogy to the case of a homo-

[3] Åke S:son Stenius, Svensk Papperstidn. **56**, 607 (1953).
[4] F. A. Steele, Paper Trade J. **100**, 37 (1935).
[5] Deane B. Judd, (with collaboration of Harrison, Sweo, Hickson, Eickjoff, Show, and Pfaffenbarger) J. Research Natl. Bur. Standards **39**, 287 (1937). Deane B. Judd, Paper Trade J. (1938), January 6.

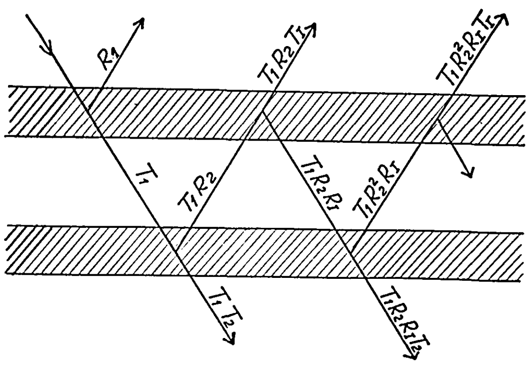

FIG. 1.

geneous specimen, for which $P=SX$. We measure only the thickness of the specimen by weight (as paper technologists usually do) and adapt the definition of S. Besides, this way of defining X and S has been anticipated by Steele[4] and by Judd[5] as far as they were applying the author's theory to paper.†

III. TWO NONHOMOGENEOUS SPECIMENS

Let us consider now two nonhomogeneous specimens of different optical properties and, perhaps, of different thickness. The character of the nonhomogeneity is of no importance. Within each specimen the functions $S=S(x)$ and $K=K(x)$ may be of any kind whatever, continuous or discontinuous. We remember that we are treating only the case in which surface reflections can be neglected. For example, we may consider the two specimens as nonhomogeneous sheets when each specimen is composed of several homogeneous sheets.

Now we measure the transmittances T_1 and T_2 and the reflectences R_1 and R_2 of both specimens in one direction (say, when the light passes from above to below) and we measure the transmittances T_I, T_{II} and the reflectances R_I, R_{II} in the opposite direction. Then we place the two specimens together and illuminate specimen 1. What will be the values of transmittance $T_{1,2}$ and reflectance $R_{1,2}$ of the combined specimen?

In Fig. 1 the path of the diffused light is symbolized by oblique beams. The light flux which hits sheet 1 is $I=1$. Firstly the portion R_1 is reflected by the first sheet and the portion T_1 passes through and hits the second sheet which reflects the portion T_1R_2 and transmits the portion T_1T_2. The portion T_1R_2 hits the lower side of the first sheet which transmits $T_1R_2T_I$ and reflects the portion $T_1R_2R_I$; from this portion the amount $T_1R_2R_IT_2$ passes through the second sheet and $T_1R_2R_IR_2=T_1R_2^2R_I$ hits again the lower side of the first sheet which transmits $T_1R_2^2R_IT_I$ and reflects again $T_1R_2^2R_IR_I=T_1R_2^2R_I^2$ to the second sheet $\cdots$ and so on, ad infinitum. Adding up the portions finally transmitted by the combined specimen, we get

$$T_{1,2}=T_1T_2(1+R_IR_2+R_I^2R_2^2+\cdots)=\frac{T_1T_2}{1-R_IR_2}. \quad (8)$$

Suming up the portions reflected by the combined specimen we get

$$R_{1,2}=R_1+T_1T_IR_2(1+R_IR_2+R_I^2R_2^2+\cdots)$$

$$=R_1+\frac{T_1T_IR_2}{1-R_IR_2}. \quad (9)$$

Since, as will be shown in the next section, $T_1=T_I$, we can write

$$R_{1,2}=R_1+\frac{T_1^2R_2}{1-R_IR_2}. \quad (10)$$

For any homogeneous sheet $R_I=R_1$. Hence for two homogeneous sheets formulas 8 and 10 reduce to

$$T_{1,2}=\frac{T_1T_2}{1-R_1R_2} \quad (11)$$

and

$$R_{1,2}=R_1+\frac{T_1^2R_2}{1-R_1R_2}. \quad (12)$$

Equations (11) and (12) can be found already in the papers of Gurevic[7] and of Zocher.[8] They are less general then Eqs. (8) and (10) because they do not hold for nonhomogeneous sheets, as do Eqs. (8) and (10).

IV. THE NONPOLARITY OF TRANSMITTANCE

We now prove the statement made in Sec. III, that

$$T_I=T_1,$$

that is to say, the transmittance has the same value, if we illuminate a nonhomogeneous sheet from one or the other side.

Transmittance of a pair of nonhomogeneous sheets is given by Eq. (8):

$$T_{1,2}=T_1T_2/(1-R_IR_2).$$

When the light passes through the pair in the inverse direction, the same equation holds. We have only to alter the indices

$$T_{II,I}=T_{II}T_I/(1-R_2R_I).$$

Dividing this equation by Eq. (8), we get

$$\frac{T_{II,I}}{T_{1,2}}=\frac{T_IT_{II}}{T_1T_2}.$$

† From measurements made by Stenius (see reference 6) can be deduced, however, that in certain types of handmade paper S_w is not exactly constant.

[6] Åke S:son Stenius, Svensk Papperstidn. 54, 663 (1951); (reference 6) 54, 701 (1951).

[7] Gurevic, Physik. Z. 31, 753 (1930).

[8] Hans Zocher, *Kolloidchemisches Taschenbuch*, (1948) third edition, p. 83.

Now we add a third sheet. While dealing with nonhomogeneous sheets, we may consider the first two sheets as one, and the third as the other one of a set of two and apply the above formula

$$\frac{T_{III,II,I}}{T_{1,2,3}}=\frac{T_{II,I}T_{III}}{T_{1,2}T_3}=\frac{T_IT_{II}T_{III}}{T_1T_2T_3}.$$

We may add a 4th, 5th, $\cdots n$th sheet, too, and find in the same way

$$\frac{T_{N\cdots III,II,I}}{T_{1,2,3,\cdots n}}=\frac{T_IT_{II}T_{III}\cdots T_N}{T_1T_2T_3\cdots T_n}.$$

Let us now increase the number n of sheets and, in order to keep the thickness of the whole set, decrease their thickness accordingly until we arrive at $n=\infty$ and infinitesimally thin sheets. We may consider such sheets as homogeneous. Therefore

$$T_I=T_1;\quad T_{II}=T_2;\quad T_{III}=T_3;\cdots T_N=T_n;\ddagger$$

and consequently

$$T_{N\cdots III,II,I}=T_{1,2,3,\cdots n}. \tag{13}$$

This means, transmittance of any nonhomogeneous specimen does not depend on the direction of illumination, in accordance with our statement.

Now we shall show that we may invert the order of the sheets (which is equivalent to inverting the direction of illumination), but we may not change the order in any other way without altering transmittance. The sheets themselves may be homogeneous or not. We take as an example a set of three different homogeneous sheets. Again, we imagine the set to be composed of a pair of sheets and a single one. We consider the pair to be a nonhomogeneous single sheet and write according to Eq. (8)

$$T_{1,2,3}=\frac{T_{1,2}T_3}{1-R_{2,1}R_3}, \tag{14}$$

substitute for $T_{1,2}$ the value of Eq. (11)

$$T_{1,2}=\frac{T_1T_2}{1-R_1R_2},$$

substitute $R_{2,1}$ from Eq. (12)

$$R_{2,1}=R_2+\frac{T_2^2R_1}{1-R_2R_1},$$

‡ We may prove this also in the following way: in the previous paper (reference 2), page 450, we find for homogeneous layers: $K=-(dT/dX)_{X\to 0}-S$ or $dT_{X\to 0}=-(K+S)dX$. This equation holds also for nonhomogeneous layers, for dK and dS would appear as products with dX only, it is to say small of the second order. Consequently, $T_{X\to 0}=-(K+S)X$, which equation is independent of the direction of illumination.

and get after some transformation

$$T_{1,2,3}=\frac{T_1T_2T_3}{1-R_1R_2-R_2R_3+R_1R_2^2R_3-R_1T_2^2R_3}. \tag{15}$$

This equation is symmetrical with regard to the 1 and 3 quantities only; we may, without altering the value of $T_{1,2,3}$, exchange T_1 for T_3 and R_1 for R_3 but not, similarly, T_2 and R_2 for T_1 and R_1 or T_1 and R_1 for T_3 and R_3. Consequently,

$$T_{1,2,3}=T_{3,2,1},$$

but

$$T_{1,2,3}\neq T_{2,3,1}$$

and

$$T_{1,2,3}\neq T_{1,3,2}.$$

V. THREE OR MORE NONHOMOGENEOUS SPECIMENS

After the manner of deriving Eq. (14), we may easily get an equation for the transmittance of three nonhomogeneous sheets

$$T_{1,2,3}=T_1T_{2,3}/(1-R_IR_{2,3}). \tag{16}$$

in which $T_{2,3}$ and $R_{2,3}$ have to be substituted by the values calculated by Eqs. (8) and (10). We also can write immediately a formula for n nonhomogeneous sheets:

$$T_{1,2,3,\cdots n}=\frac{T_1T_{2,3,4,\cdots n}}{1-R_IR_{2,3,4,\cdots n}}. \tag{17}$$

In the same way we may write a formula for reflectance:

$$R_{1,2,3\cdots n}=R_1+\frac{T_1R_{2,3,4\cdots n}}{1-R_IR_{2,3,4,\cdots n}}. \tag{18}$$

Both equations, of course, may be used for sets of homogeneous sheets too. (Then we do not need to distinguish between R_I and R_1)

VI. NONHOMOGENEOUS SPECIMENS WITH BACKING

Paper and paint technologists frequently determine the reflectance of a specimen upon a backing from which the transmitted light is reflected back to the specimen by a surface of reflectance $r\neq 0$.

In Eq. (10) we may consider the second sheet as backing ($R_2=r$). So we get immediately an equation representing reflectance $R_{1(r)}$ of a nonhomogeneous specimen with backing:

$$R_{1(r)}=R_1+\frac{T^2r}{1-R_Ir}. \tag{19}$$

(In spite of the fact that there is now a single sheet only, we keep the indices 1 and I in order to mark the direction of the illumination.) Solving for T from

Eq. (19) leads to the formula

$$T=[(R_{1(r)}-R_1)(1/r-R_I)]^{\frac{1}{2}}, \tag{20}$$

which permits one to calculate the transmittance of a nonhomogeneous specimen from no other measurements than of reflectances. Formulas 19 and 20 correspond with formulas (37) and (37a) derived in the previous paper[2] for a homogeneous specimen. They become identical, if we assume $R_I=R_1$, as we have to, in the case of homogeneity.

We remember that T is not altered by turning the specimen over upon the backing, and therefore we may write also

$$T=[(R_{I(r)}-R_I)(1/r-R_1)]^{\frac{1}{2}}. \tag{21}$$

From Eqs. (20) and (21) we may eliminate T and get a relation between the different kinds of reflectance of a nonhomogeneous specimen:

$$\frac{R_{1(r)}-R_1}{1-rR_1}=\frac{R_{I(r)}-R_I}{1-rR_I}. \tag{22}$$

Now it would be very interesting to get formulas corresponding to those derived by Judd,[5,9] connecting reflectances of the same specimen in the cases of perfectly white ($r=1$; $R_{(r)}=R_{(1)}$), gray or not perfectly white ($r=r$; $R_{(r)}=R_{(r)}$) and perfectly black backing ($r=0$; $R_{(r)}=R_{(0)}\equiv R$). We substitute $r=1$ and $R_{1(r)}=R_{1(1)}$ in Eq. (20):

$$T=[(R_{1(1)}-R_1)(1-R_I)]^{\frac{1}{2}}, \tag{23}$$

and combine this equation with the original Eq. (20). We get the formula

$$R_{1(1)}=\frac{R_{1(r)}-R_1+r(R_1-R_{1(r)}R_I)}{r(1-R_I)} \tag{24}$$

which corresponds to formula (36a) of the previous paper.[2] By introducing contrast ratios

$$C_{1(r)}=R_1/R_{1(r)}$$

and

$$C_{1(1)}=R_1/R_{1(1)},$$

we get

$$C_{1(1)}=\frac{rC_{1(r)}(1-R_I)}{1-rC_{1(r)}-C_{1(r)}-rR_I}. \tag{25}$$

This formula corresponds to formula 2b of Judd[5] and simplifies to it, if we put $R_I=R_1\equiv R$. If the specimen is of the type treated in Sec. II ($K/S=\text{const.}$), then $R_I=R_1$ and Judd's formula holds. Judd, when stating that his formula applies to nonhomogeneous specimens, certainly had in mind this special case of nonhomogeneity.

[9] Deane B. Judd, J. Research Natl. Bur. Standards **12**, 345 (1934) (reference 9); **13**, 281 (1934).

VII. PSEUDOHOMOGENEOUS LAYERS

In the case $a\equiv(S+K)/S=\text{const}$, $S=S(x)$, which was treated in Sec. II, not only transmittance, but also reflectance is independent of the direction of illumination, so that $R_1=R_I$. We may prove it e.g. by Eq. (32) of the previous paper,[2] which we may write as

$$R=a-(T^2+a^2-1)^{\frac{1}{2}}.$$

a is constant, therefore T being always independent of the direction of illumination, R has to be independent too.

From this follows that in Secs. 3–6 we can set $R_I=R_1$ in all equations and so use the simpler formulae, valid for homogeneous layers.

There is another case, where we can do so: when $a\equiv(S+K)/S$ is not constant, but symmetrically distributed in the layer, that is to say when

$$a(x)=a(X-x).$$

Then there is no difference between the two directions and consequently also $R_I=R_1$. Stenius[6] when applying the author's theory on homogeneous layers[1,2] used this relation. He was working with somewhat nonhomogeneous paper sheets. In order to make them behave like homogeneous ones, he folded them so that the folded sheets became symmetrical.

EXPERIMENTAL PART

An interesting result of our theory is the fact that transmittance of a nonhomogeneous specimen does not depend on the direction of illumination (whereas reflectance and absorption may be altered considerably, if we invert the direction of illumination). This result has been verified experimentally.

The experiments were made (1) with fairly white, (2) with pale blue, and (3) with intense blue paper. The white paper was commercial filter paper, the two blue ones have been prepared by dyeing the same paper with methylene blue.

The measurements were made with the aid of a simple selenium cell photometer and with diffused illumination. The photometer was connected with a Multiflex galvanometer. One division of the graduation of the galvanometer corresponded to approximately 0.03 percent of the full illumination. Absolute exactitude was not considered to be important, because relative exactitude was sufficient for our purpose.

Four series of experiments have been made with different numbers and combinations of the three kinds of paper. In every series the combination concerned has been permutated thoroughly as to the order of the different sheets. Every series has been measured three to six times in order to get fair average values.

The results are compiled in Table I. The table shows, in full accordance with the theory, that when the order of the sheets is only inverted, but otherwise kept, the deviations of the measured transmittances do not surpass the limits of experimental errors. On the other hand, any other permutation of the order of sheets alters transmittance perceptibly, sometimes very considerably (see, e.g., series 2). The lowest values of transmittance have been obtained when the darkest sheet was placed into the middle of the set. Also this result accords with the theory. In order to demonstrate it, we transform Eq. (15) and get

$$T_{1,2,3}=\frac{T_1T_2T_3}{1-(R_1R_2+R_2R_3+R_1R_3)+R_1R_2R_3\cdot(1+R_2^2-T_2^2)/R_2}. \quad (26)$$

This equation is symmetrical with respect to the three sheets, with exception of the last factor of the last term in the denominator,

$$(1+R_2^2-T_2^2)/R_2.$$

Therefore, when the arrangement of the sheets is changed, this factor alone is responsible for the difference in transmittance. This factor, however, depends on the darkness of the sheet placed in the middle. According to the previous paper,[2] Eq. (33), the factor

$$(1+R_2^2-T_2^2)/R_2=2a_2=1/R_{\infty 2}+R_{\infty 2},$$

$R_{\infty 2}$ being the reflectivity of the sheet 2 (≡reflectance of an infinitely thick pile of sheets having the optical qualities of sheet 2). The smaller $R_{\infty 2}$ the higher is $2a_2$ and therefore the denominator in Eq. (26), that is to say, the darker the middle sheet, the smaller is the transmittance $T_{1,2,3}$ of the whole system.

TABLE I.

Series	Arrangement of sheets	Results (galvanometer reading)	Average
1	3-1-1-1-1	23.9; 24.2; 23.3	23.8
	1-1-1-1-3	26.7; 23.0; 23.0	24.2
	1-1-1-3-1	16.6; 17.5; 16.9	17.0
	1-3-1-1-1	16.9; 17.0; 16.9	16.9
	1-1-3-1-1	16.2; 15.7; 15.2	15.7
2	3-1-1	75.5; 73.0; 71.3; 72.9	73.2
	1-1-3	74.3; 73.0; 71.5; 71.1	72.5
	1-3-1	55.0; 54.9; 54.2; 54.0	54.5
3	1-2-3	43.1; 43.3; 42.2	42.9
	3-2-1	43.5; 43.6; 42.8	43.3
	2-3-1	40.3; 38.3; 37.9	38.8
	1-3-2	39.3; 38.3; 37.3	38.3
	2-1-3	45.2; 45.6; 45.5	45.4
	3-1-2	45.3; 45.2; 45.4	45.3
4	2-2-3	34.3; 35.5; 34.5; 33.7; 33.7; 33.0	34.1
	3-2-2	34.7; 34.4; 34.5; 34.1; 33.8; 33.5	34.2
	2-3-2	32.2; 31.1; 32.2; 30.6; 30.2; 30.5	31.1

SUMMARY

(1) Transmittance and reflectance of nonhomogeneous turbid systems can be calculated in ways analogous to those used in the preceding paper[2] for homogeneous systems. There are special cases of practical importance that can be treated quite easily.

(2) In nonhomogeneous systems reflectance depends in general on the direction of illumination, whereas transmittance is always independent of it.§

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§ In a discussion in the session of the Brazilian Academy of Science of December 28, 1949, Mr. Hans Zocher pointed out that this result is a special case of his more general principle of apolarity of energy transmission. In a paper to be published briefly, Hans Zocher and C. Török will show that this principle can be deduced of their Space-Time-Asymmetry concept. [See also Zocher and Török, Proc. Natl. Acad. Sci. (U.S.A.) **59**, 681 (1953)].