Natural Phenomena and Special Effects

Real-time simulation of watery paint

By Tom Van Laerhoven* and Frank Van Reeth

Existing work on applications for thin watery paint is mostly focused on automatic generation of painterly-style images from input images, ignoring the fact that painting is a process that intuitively should be interactive. Efforts to create real-time interactive systems are limited to a single paint medium and results often suffer from a trade-off between real-timeness and simulation complexity. We report on the design of a new system that allows the real-time, interactive creation of images with thin watery paint. We mainly target the simulation of watercolor, but the system is also capable of simulating gouache and Oriental black ink. The motion of paint is governed by both physically based and heuristic rules in a layered canvas design. A final image is rendered by optically composing the layers using the Kubelka–Munk diffuse reflectance model. All algorithms that participate in the dynamics phase and the rendering phase of the simulation are implemented on graphics hardware. Images made with the system contain the typical effects that can be recognized in images produced with real thin paint, like the dark-edge effect, watercolor glazing, wet-on-wet painting and the use of different pigment types. Copyright © 2005 John Wiley & Sons, Ltd.

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Introduction

Creating a digital equivalent of the traditional painting process has several advantages. The possibility of experimenting with various techniques and painting media with control over aspects, such as drying time, undoing mistakes, saving intermediate results, and the ability of introducing a wide range of digital tools, makes a painting system a valuable tool for both novices and experienced artists.

Due to the complexity of this process, however, it is a challenging task to simulate all this in real time. Existing work reveals that numerous problems remain in visual results, as well as in the creational process itself. In both Western and Oriental versions of digital painting quite realistic results can be obtained, but most often at the expense of a tedious, non-intuitive way of creating them. User input and rendering usually occur in separate stages of the simulation process, creating a mismatch between what the system delivers and what the artist actually had in mind. Our work targets precisely these problems.

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Figure 1. A computer-generated watercolor image.
Main Contributions

We introduce a new canvas model for the interactive simulation of thin, watery paint in real time. It is the first model that provides real-time painting experience with watercolor paint, while comprising sufficient complexity to capture its complicated behavior, its interactions with the canvas as well as its chromatic properties. We extend our previous work on a parallel implementation\(^1\) with a new approach, suitable for graphics hardware, the Kubelka–Munk diffuse reflectance model and the capability to produce paintings with paint media related to watercolor, like gouache and Oriental black ink.

Background

Painting systems that capture the complexity, variety, and richness of painting media only recently began to appear in literature. Cockshott identifies the main problem of existing painting systems as being the lack of sparkle in the images they produce when compared to those made by traditional methods and media.\(^2\) He claims this is caused by the shallowness of painting models and the lack of understanding the process of real painting and the behavior of paint. His own model, based on a cellular automaton, therefore includes rules accounting for surface tension, gravity, and diffusion. At the same time, Small introduces a canvas model for watercolor, again based on the cellular automaton principle.\(^3\) Prior to their work, the actual painting process was mostly limited to the rendering of a brush imprint, with notable results by Greene’s drawing prism and Strassmann’s hairy brush.\(^4,5\)

Curtis et al.\(^6\) adopt a more sophisticated paper model and a complex shallow layer simulation for creating watercolor images, incorporating the work on fluid flows by Foster and Metaxas.\(^7\) The painting consists of an ordered set of translucent glazes or washes. The individual glazes are rendered and composed using the Kubelka–Munk equations to produce the final image.\(^8\) Their model is capable of producing a wide range of effects from both wet-in-wet and wet-on-dry painting. Fluid flow and pigment dispersion are again realized by means of a cellular automaton. In fact, cellular automata\(^9\) and related techniques play an important role in the work of many authors.\(^2,6,10,11\)

All of the above work, however, is more related to automatic rendering than to the interactive painting experience, mostly because the computational complexity did not allow real-time processing.

Baxter et al. are the first to present a fully interactive physically-based paint simulation for thick oil-like painting medium.\(^12,13\) The IMPaSTo application is a mature painting system, exploiting graphics hardware for both the physical simulation of paint flow and the rendering with an interactive implementation of the Kubelka–Munk diffuse reflectance model. As the system is using an advection scheme to model the paint dynamics it neglects ambient behavior of the paint medium like diffusing, running, and dripping effects, which is our main objective in this paper.

The possibility to create watercolor images is also present in several commercial painting systems. Most significant is Corel Painter IX, which incorporates re-touchable wet areas and lets users control the diffusion process.\(^14\)

Although closely related in that they both use brush and water, a distinction can be made in the goals pursued by Oriental black ink paintings and Western watercolor paintings. The former uses highly absorbent, thinner, and more textured paper types. Other apparent differences can be found in the brush types and paint techniques. An extensive comparison of both techniques is given by several authors.\(^15,16,17\)

Overview and Architecture

The canvas model has a layered design, consisting of three active layers and an unlimited number of passive layers. The passive layers are considered to contain previously drawn strokes that have dried and no longer participate in the simulation, except in the final step when the canvas is rendered. The part of the simulation that handles the dynamics of pigment and water takes place in the active layers. Inspired by the three-layer canvas model of Curtis et al.,\(^5\) our active layers have very similar tasks. The underlying computational model, however, is very different (Figure 2). The motivation for using this three-layer design stems from an analysis of the behavior of paint; three different states for pigment or water can be distinguished:

- Pigment and water in a shallow layer on top of the canvas.
- Pigment deposited on the canvas.
- Water absorbed by the canvas.

In what we will be referring to as the shallow fluid layer, a 2D fluid body consisting of a mixture of water and paint pigment represents a stroke on top of the canvas. The water will eventually evaporate or be
absorbed into the canvas capillary layer, while pigment particles will settle within the irregularities of the canvas surface, represented by the surface layer. Figure 2 presents a schematic view of the canvas model. In subsequent sections we will elaborate on the details of each layer, starting with the active layers.

**Shallow Fluid Layer**

In its initial form, a stroke consists of a shallow layer of fluid that resides on top of the canvas surface. The movement of the fluid flow through time is given by the two-dimensional Navier—Stokes equation. It allows us to take larger time steps and results in a faster simulation that never ‘blows up.’ Baxter et al. use a similar approach in their viscous paint application.

The state of a vector field \( \vec{v} \) defining the velocities of a fluid body at any given time and space during simulation is given by Equation (1), which is a variant of the Navier–Stokes equation:

\[
\frac{\partial \vec{v}}{\partial t} = - (\vec{v} \cdot \nabla) \vec{v} + \nu \nabla^2 \vec{v}
\]  

In order to adapt this procedure for the purpose of moving paint fluid, we need to introduce the following variables for this layer:

- A vector field \( \vec{v} \) defining the fluid velocity. In discrete form we assign to the center of each cell \( (i,j) \) velocity \( (v_x, v_y)_{i,j} \).
- Water quantity \( w_{i,j} \) for each cell, measured in terms of height, and constrained within \( [w_{\text{min}}, w_{\text{max}}] \).
- Amount of pigment \( (p_{dx})_{i,j} \) for each cell, as a fraction of the cell surface. Each pigment type is denoted by a unique index.
- A diffusion constant \( \nu \), determined by the ratio of mass density \( \rho \) and viscosity \( \eta \) of the fluid.

Given these variables, we can mainly follow the method of solution described by Stam to update the state of a fluid flow. As shown in the next sections, some modifications were made that are specific to our problem.

A time step in this part of the simulation requires four operations:

1. Add water, pigment, and velocity values.
2. Update velocity field \( \vec{v} \) (Equation (1) and section).
3. Update water quantities \( w \) (Equation (2) and section).
4. Update pigment quantities \( p_{dx} \) for each pigment (Equation (3) and section).

Once the velocity field is updated according to Equation (1), we can use it to update each cell’s water quantity (Equation (2)) and pigment quantity for each pigment (Equation (3)):

\[
\frac{\partial w}{\partial t} = - (\vec{v} \cdot \nabla)w + \nu w \nabla^2 w
\]

\[
\frac{\partial p_{dx}}{\partial t} = - (\vec{v} \cdot \nabla)p_{dx} + \nu p \nabla^2 p_{dx}
\]
Updating the Velocity Vector Field

All steps in the update velocity routine are enumerated in Table 1. The ‘addHeightDifferences’ step accounts for differences in water height and will be explained in the next section.

The state of a two-dimensional fluid flow at a given instant of time can be modeled as a vector field that is sampled at the center of each cell of a 2D grid. Updating the velocity according to Equation (1) now equals resolving the two terms that appear at the right-hand side.\[^\text{20}\]

1. self-advection \(-\vec{\sigma} \cdot \nabla \vec{\sigma}\)
2. diffusion \(\nu \nabla^2 \vec{\sigma}\)

Self-Advection

Self-advection calculates how the values in the velocity field affect the velocity field itself. In Reference,\[^\text{20}\] an implicit method is described that assigns a particle to each cell center, conveying that cell’s velocity value. Intuitively, the particles are dropped in the velocity field and re-evaluated at the resulting position.

Diffusion

Diffusion of the velocity field, caused by the second term at the right-hand side of equation 1, accounts for spreading of velocity values at a certain rate. We use the Jacobi method to solve this problem, which takes the form of a Poisson equation, in order to find \(\vec{v}_{\text{new}}\).

Updating Water Quantities

The previous section handled the computation of the velocity field for this time step. We will now use this new vector field to update the scalar field of water quantities. Again it means first adding additional water quantities \(w_{\text{source}}\) to the scalar field and then solving the two terms at the right-hand side of the equation:

- diffusion \(\nu w \nabla^2 w\)
- advection \(- (\vec{\sigma} \cdot \nabla)w\)

In practice, the same methods from the previous section could be used at this point. We will develop our own algorithms for diffusing and advecting the water, however, because we want to add constraints on the upper and lower boundaries of a cell’s water content, and we want a mechanism to simulate the ‘dark edge’ effect (Figure 4(k)).

Water Diffusion

For the diffusion process of water quantities we first annotate the velocity field with additional diffusing motion by accounting for differences in water quantities between neighboring cells. This is the previously unmentioned ‘addHeightDifferences’ step in Table 1.

The water quantity of a cell in the shallow layer is expressed in terms of water height. During the diffusion process, water needs to reach a point at which each cell contains an equal water height. Therefore, we calculate the velocities that are necessary to obtain equal heights in all cells. The resulting vector field \(\vec{v}_{\text{h}}\) is combined with the velocity field \(\vec{v}_{\text{old}}\) we already calculated:

\[
\vec{v}_{\text{new}} = \omega_{\text{i}} \vec{v}_{\text{old}} + \omega_{\text{h}} \vec{v}_{\text{h}},
\]

where \(\omega_{\text{i}}\) and \(\omega_{\text{h}}\) are weight factors and \(\omega_{\text{i}} + \omega_{\text{h}} = 1\). In all examples we used \(\omega_{\text{h}} = 0.06\). The advantage of this approach is that the movement of pigment, which depends on the velocity field, will also be affected by differences in water quantities. This way we can obtain the ‘dark edge’ effect, a result of the fact that at the edge of a stroke water evaporates faster and is replaced by water and pigment from the interior of the stroke. Figure 4(k) shows an example of a computer-generated brush strokes with dark edges.

Water Advection

For each cell, we measure the volume of water that is exchanged with all neighboring cells. As an example,
we will calculate the amount of water that flows from the center cell to its right neighbor, with velocities \( \vec{v}_{\text{center}} \) and \( \vec{v}_{\text{right}} \) respectively, as shown in Figure 3(a):

\[
\vec{v}_{\text{average}} = \frac{\vec{v}_{\text{center}} + \vec{v}_{\text{right}}}{2}
\]  

Any point along this border has velocity \( \vec{v}_{\text{average}} \) and travels within a given time step \( \Delta t \) a distance \( \Delta x = (\vec{v}_{\text{average}}) \cdot \Delta t \) in horizontal direction. Therefore, the area covered by the volume of displaced water is given by the colored area in Figure 3(a), and equals \( \Delta x \) (cell height).

Finally, from the amount of water in the cell, the total volume of displaced water \( \Delta V = \Delta x \) (cell height)\( m_{ij} \) can be determined. The change in water quantity is given by Equation (5), and is also shown in Figure 3(b)

\[
\Delta w = \frac{\Delta V}{(\text{cell width} \times \text{cell height})}
\]  

The same procedure is followed to calculate the exchanged water quantities with the three remaining neighbors. We add up the results and divide it by four, because each neighbor contributes exactly one quarter to the total flux. Constraining the cell’s upper and lower water quantities can be done by simply clamping all individual exchanged volumes of water. It also ensures conservation of mass.

**Evaporation of Water in the Shallow Fluid Layer**

This is modeled by removing at each time step a volume of water \( \Delta V_{\text{top}} = \varepsilon_{\text{shallow}} \Delta t (\text{cell width} \times \text{cell height}) \), according to the cell’s water surface and the evaporation rate \( \varepsilon_{\text{shallow}} \).

Evaporation also occurs at the sides of cells that have neighbors without water. This way we incorporate the fact that water evaporates faster at the edges of a stroke.

**Updating Pigment Concentrations**

The velocity field not only causes movement of water but also governs the movement of pigment concentrations. In this last stage the changes in pigment concentrations will be calculated.

Table 3 gives the three basic steps taken when moving pigment concentrations in the shallow fluid layer according to a given velocity field. It shows that this procedure is similar to moving water quantities, adding a scalar field of new pigment quantities \( p_{\text{idx}, \text{source}} \) and solving the two terms in Equation (3):

- diffusion \( \nu_p \nabla^2 w \)
- advection \( -(\vec{v} \cdot \nabla)p \)

The pigment source term is comprised of amounts of pigment that are added by a brush.
The Diffusing Process of Pigment Concentrations

This again uses the Jacobi method for calculating the updated scalar field $p_{\text{dx,new}}$ of each pigment:

$$(1 - \nu \Delta t \nabla^2) p_{\text{dx,new}} = p_{\text{dx,old}}$$

with $\nu$ indicating the pigment diffusion rate.

Advection of Pigment

Advection of pigment or the movement of pigment caused by the velocity vector field relies on the algorithm for moving water. The outgoing fraction of pigment for a similar situation to the one depicted in Figure 3(a) is:

$$\Delta p_{\text{dx}} = \frac{\Delta t (\text{cell height}) p_{ij}}{\text{(cell width x cell height)}}$$

Boundary Conditions

One issue we did not consider so far is the fact that the movement of paint must respect the boundaries of a stroke. In our case, a boundary is defined as the interface between the paint and the atmosphere. The boundaries can move, however, as the stroke expands through capillary activity, as we will discuss later.

At any given time step, no pigment or water is allowed to travel across these boundaries. Fortunately, our water and pigment advection algorithms implicitly guarantee this condition. Both algorithms define at each cell the movement of substance to neighboring cells. If we know which cells belong to the stroke, we can simply check if a neighboring cell lies within the stroke and is allowed to receive material. Another consequence of a boundary is that it influences the velocity vector field, making fluid flow along it. This is done by setting the

**Table 3. Updated pigment quantities**

updatePigmentQuantities (p, source, dt)
addPigment (p, source)
diffusePigment (p, dt, diff_rate)
advectPigment (p, dt)
normal component of the velocity vector at boundary
cells to zero.\textsuperscript{7,20}

**Surface Layer**

Previous sections discussed the activity of water and
pigment at the shallow layer. The pigment is initially
dropped in the shallow fluid layer, but eventually ends
up being deposited on the surface of the paper canvas.
In the meantime, there is a continuous transfer of pig-
ment between the shallow fluid layer and the surface
layer (Figure 5).

The surface layer keeps track of the deposited
amounts of pigment \( \rho_{idx} \) for each cell \((i,j)\).

Pigment will be adsorbed and desorbed according to
Equations (7) and (8) respectively:

\[
\Delta p_{ad} = \Delta t (p_{water}(1.0 - w_{frac})(1.0 - h)\gamma_{idx}) \delta_{idx} \quad (7)
\]

\[
\Delta p_{de} = \Delta t (p_{dep}w_{frac}(1.0 - (1.0 - h)\gamma_{idx}) \delta_{idx} \omega_{idx}) \quad (8)
\]

Both equations depend on the amount of water \( w_{frac} \)
in the shallow layer as a fraction of the maximum water
allowed, the paper height fraction \( h \) at that cell, and
several properties of pigment \( idx \):

- Pigment granulation \( 0 \leq \gamma_{idx} \leq 1 \).
- Pigment density \( 0 \leq \delta_{idx} \leq 1 \).
- Pigment staining power \( 0 \leq \omega_{idx} \).

The granulation factor determines the influence of the
paper height on the amount of pigment transfer. Pig-
ment with high granulation will settle more easily in the
cavities of the paper canvas. A high density factor
results in pigment that is deposited more quickly. The
staining power defines the pigment’s resistance of being
picked back up by the shallow water layer. Taking these
observations into account, we end up with Equations (7)
and (8). A similar transfer algorithm was used by
Reference [6], but ignored the effect of water quantity.

**Capillary Layer**

The capillary layer represents the inner paper structure.
Within the simulation it is responsible for the movement
of absorbed water, allowing a stroke to spread across its
original boundaries.

At this point in the simulation, water movement is
governed by microscopic capillary effects that can be
described in terms of diffusion. The paper structure is
represented as a two-dimensional grid of cells or ‘tanks’
that exchange amounts of water. If each cell
has a different capacity, water will spread irregularly
through the paper, which is the behavior we want to
model. The first thing we have to do is to assign
capacities to each cell by generating a paper-like
texture.

**Fiber Structure and Canvas Texture**

The structure of the canvas affects the way fluid is
absorbed and diffused in the capillary layer. The canvas
texture should also influence the way pigment is trans-
ported and deposited. Canvas typically consists of an
irregular adsorbent fiber mesh, with the spaces between
fibers acting as capillary tubes to transport water. We
use the algorithm described by Worley in Reference [22]
to produce a textured surface. The same strategy was
used by several authors.\textsuperscript{6,23} It creates a procedural
texture that can serve as a height field (Figure 5).

**Capillary Absorption, Diffusion and
Evaporation**

The water of a brush stroke will gradually be absorbed
by the paper canvas. At each time step an amount of
water \( \Delta w = \alpha \Delta t \) is transferred from the shallow fluid
layer to the capillary layer, determined by the rate of
absorption \( \alpha \). This amount is clamped according to the
amount of water left in the shallow fluid layer, and the
available capillary space.

Water in the capillary layer moves to neighboring
cells through a diffusion process like we described in
context of the shallow fluid layer, and disappears by
evaporation at a rate \( \epsilon_{capillary} \). The evaporation process
removes each time step an amount of water
\[ \Delta w = \epsilon_{\text{capillary}} \Delta t \] from every cell in the capillary layer that has no water left in the shallow fluid layer above.

**Graphics Hardware Implementation**

All algorithms we described so far were implemented on graphics hardware as fragment shaders using NVIDIA’s high-level shading language Cg. The simulation loop relies on the framebuffer-object extension, allowing the results of a rendering pass to be stored in a target texture. Each operation in Tables 1, 2, and 3 is mapped to one or more fragment programs.

Texture objects with floating-point precision were created for each of the following data sets: 2D velocity vectors, water and pigment quantities from both the shallow fluid and the surface layer, water quantities in the capillary layer, and the canvas texture and its reflection coefficients. A total of four textures carry the pigment concentrations in both the shallow fluid layer and the surface layer, so the current implementation limits the number of pigments in each active cell to eight.

The above list excludes several textures that were used to store intermediate results. Several optimizations were made, including the reduction of texture look-ups by caching wet neighbors of a cell, and making sure only relevant parts of the canvas are updated by using an overlay grid that keeps track of areas that were touched by the brush. Just these modified sub-textures are processed in the next time step. Each tile is also annotated with a time-to-live value, based on a fair estimation of the paint’s time to dry.

The brush is implemented as a fragment shader that writes pigment, water and velocity values in the appropriate textures according to user input.

**Results**

All results are created with our application on an Intel(R) Xeon(TM) 2.40 GHz system equipped with a NVIDIA GeForce FX 6800 graphics card. A Wacom tablet interface was used as a brush metaphor. In all cases the canvas measured 800 \times 600 cells, with an overlay grid of 32 \times 32 tiles. The frame rate of 20 frames/second is affected when a user covers a very large area within the same active layer and draws fast enough so that the drying process does not deactivates any tiles in the overlay grid. In this situation, user interaction is still possible at about half the normal frame rate. Users are provided with an intuitive interface, displaying the canvas and a default palette with 12 different pigment types. Basic operations include the possibility to save intermediate results, and canvas operations like drying, clearing, and starting a new layer.

**Watercolor**

The strokes in Figure 4 show examples of typical watercolor paint effects. Several images created with our system are depicted in Figure 6.

**Oriental Black Ink**

Although the brushes and techniques used in Oriental paintings are very different from those in Western painting, the mechanics of pigment and water are quite similar. The canvas is generally more textured and more absorbent, and the dense black carbon particles are smaller and able to diffuse into the paper. The former property is easily obtained in our simulation by generating a rougher canvas texture and using a higher absorption constant. Despite the fact that our canvas model does not simulate pigment particles inside the canvas structure, ink diffusion can still be handled by the top layer and produce the typical feathery pattern. The palette consists of very dark pigment with high...
density. Figure 7 depicts a computer-generated painting in black ink, compared with the original ‘La Mort’.

**Gouache**

Gouache is watercolor to which an opaque white pigment has been added. This results in stronger colors than ordinary watercolor. A layer of paint covers all layers below, so paint is not applied in glazes. Also, gouache is not absorbed in the canvas but remains on the surface in a thick layer, creating flat color areas. These properties can be mapped to our model by replacing the KM optical model with a simpler algorithm that blends layers together based on local pigment
concentrations. Using a higher viscosity factor and loading more pigment in the brush results in thicker paint layers. Figure 8 shows an example of computer-generated gouache.

**Conclusion and Future Work**

In this paper we presented the results of our research on a physically-based system for creating images with watery paint. The goal was to design a system that runs in real time, and yet able to recreate the various effects specific to this medium. We implemented a prototype on graphics hardware using several fragment programs that operate on simulation data stored in texture objects. Results show that a wide range of effects can be achieved. The interactive process of creating them was positively evaluated by several users. Although most users did not indicate the currently used simple brush model as a major shortcoming, future work includes the design of a better brush model to produce more realistic stroke shapes. Current work also includes the creation of animated paintings as well as the design of several tools that are quite common in real painting but have no digital counterpart yet.

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